problem of this type, consider the simply supported beam shown in Fig. 1. The cross-spectral density for the generalized forces in modes j and k is[8]

$$F_{jk} = \int\int S_{PP}(x_1, x_2; \omega)\, \phi_j(x_1)\, \phi_k(x_2)\, dx_1\, dx_2$$

where $\phi_j(x)$ are the orthogonal modes for the original structure. For a spatially uncorrelated, weakly stationary pressure loading, the matrix for cross-spectral densities for the generalized forces becomes

$$\lceil F_{jj}(\omega) \rfloor$$

where

$$F_{jj}(\omega) = \left[\int \phi_j(x)\, dx\right]^2 S_{PP}(\omega)$$

The cross-spectral density matrix describing the modal response of the simply supported beam is

$$\lceil S^o_{q_j q_j}(\omega) \rfloor = \lceil H_o{}^*(\omega) \rfloor \lceil F_{jj}(\omega) \rfloor \lceil H_o(\omega) \rfloor$$

where $\lceil H_o(\omega) \rfloor$ is the matrix of frequency response functions describing the modal response of the original structure.

Now consider the addition of the elastic support with stiffness equal to K_s. The support is located at $x = x_o$. Thus the constraint relation is

$$f_1 = w(x_o, t) - p_1 = 0$$

where $w(x, t)$ is the beam vertical deflection and p_1 is the displacement of the elastic support at its beam attachment point. From the expression

$$w(x, t) = \sum_{i=1}^{n} \phi_i(x)\, q_i(t)$$

we see that

$$f_1 = \sum_{i=1}^{n} \phi_i(x_o)\, q_i(t) - p_1 = 0$$

Therefore the constraint parameters in Eq. (15) are

$$\beta_j = \phi_j(x_o) \qquad j = 1, 2, \ldots, n$$

Also, $\{\lambda\} = \lambda_1$ since there are no added rigid constraints and only one elastic constraint. Therefore, Eq. (3) becomes

$$K_s p_1 = -\lambda_1$$

and $\lceil T \rfloor$ reduces to $T = K_s{}^{-1}$. $[R]$ reduces to the scalar function of frequency

$$R(\omega) = \sum_{j=1}^{n} \phi_j{}^2(x_o)\, H_j(\omega) + K_s{}^{-1}$$

Evaluating the cross-spectral density matrix for modal response yields, after extensive algebraic manipulation

$$S_{q_s q_s}(\omega) = \left\{ S^o_{q_s q_s}(\omega) \left| \sum_{j \neq s}^{n} \phi_j{}^2 H_j{}^* + K_s{}^{-1} \right|^2 + |H_s|^2 \phi_s{}^2 \sum_{l \neq s}^{n} \phi_l{}^2 S^o_{q_l q_l}(\omega) \right\} \Big/ |R(\omega)|^2$$

for $t = s$, and

$$S_{q_s q_t}(\omega) = \phi_s \phi_t \left\{ H_s{}^* H_t \sum_{j \neq s,t}^{n} \phi_j{}^2 S^o_{q_j q_j}(\omega) - S^o_{q_s q_s}(\omega)\, H_t \times \left(K_s{}^{-1} + \sum_{j \neq s}^{n} \phi_j{}^2 H_j{}^* \right) - S^o_{q_t q_t}(\omega)\, H_s{}^* \left(K_s{}^{-1} + \sum_{j \neq t}^{n} \phi_j{}^2 H_j \right) \right\} \Big/ |R(\omega)|^2$$

for $t \neq s$ and for notational convenience, the terms of the diagonal matrix $\lceil H_o(\omega) \rfloor$ have been written H_j, and $\phi_i(x_o)$ has been written ϕ_i. Notice that the right-hand sides of these equations contain only those terms which characterize either the constraint parameters, or the system and its response prior to constraint addition. Those terms which characterize the original system and its response could easily be stored during an initial computer analysis and utilized later to investigate constraint modification.

Conclusions

An expression has been given for computing the matrix of cross-spectral density functions for a system which is subjected to constraint modification. Second-order response statistics for the displacement of the modified structure could be determined in the usual manner once the appropriate cross-spectral density matrix has been generated. Although not shown explicitly in this Note, it appears that the technique can be generalized to investigate system modification.

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Flutter of a Panel Supported on an Elastic Foundation

INDERJIT CHOPRA*

National Aeronautical Laboratory, Bangalore, India

Introduction

A PANEL supported on an elastic medium often finds application in the construction of aerospace structures. This elastic medium can be any springy material such as a heat shield or damping tape attached to one side of the panel; the other side of the panel is exposed to the air flow. Dugundji, Dowell, and Perkin[1] have analyzed the flutter of a long panel resting on a linear elastic foundation in subsonic flow using the theory due to Miles.[2] They found that flutter is possible for a particular panel configuration in subsonic flow, and this flutter is mainly of a traveling-wave type. These results were also confirmed by experiment. Johns[3] studied the static aeroelastic instability of orthotropic rectangular panels resting on an elastic medium for low speed as well as supersonic flows. He has shown that the elastic medium affects significantly the aeroelastic divergence of a panel in incompressible flow. McElman,[4] as well as Johns and Taylor[5] investigated the flutter behavior of two flat panels connected by a linear elastic medium, where the flow is exposed to only one surface of the construction. They found that the response of the elastic medium, in conjunction with inplane forces in the panels, strongly affects the flutter characteristics. Dugundji,[6] who has made an exhaustive analysis of the

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* Scientist, Structural Sciences Division. Presently Graduate Student in the Department of Aeronautics and Astronautics, Massachusetts Institute of Technology.

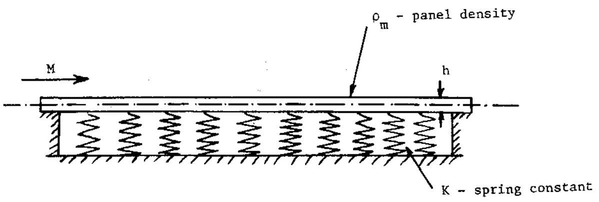

Fig. 1 A finite panel resting on elastic springs.

effect of aerodynamic damping on panel flutter at supersonic Mach numbers for various configurations, has also discussed the problem of a panel resting on an elastic foundation.

In this paper, the flutter behavior of a thin panel resting on a linear elastic foundation is studied, using an analogy with the flutter of a panel of the same geometry without spring support.

Analysis

Consider a flat panel of finite dimensions exposed to supersonic flow on its upper surface and to still air on the other side (Fig. 1). The panel rests on an elastic foundation, which is mathematically idealized as closely spaced linear springs. Using quasi-steady theory for the aerodynamic loading, Dugundji[6] has made an exact linear analysis of the equations representing flutter of such configurations. The critical flutter condition occurs when the total damping of the system becomes zero, and the results can be seen from Eqs. (26) and (27) of Ref. 6,

$$Q_I/(-Q_R)^{1/2} = g_T \tag{1}$$

and

$$(-Q_R)^{1/2} = \omega_{F_e} \tag{2}$$

All the above parameters are defined in Ref. 6. Here Q_R and Q_I refer to the composite system and $\bar{Q}_R$ and $\bar{Q}_I$ represent the system without the springs; it can be seen that $\bar{Q}_R = Q_R + k$ and $\bar{Q}_I = Q_I$. Placing these parameters into the above equations and rearranging, one may obtain

$$g_{T_e} = g_T(1+\gamma)^{1/2} \tag{3}$$

$$\omega_{F_e} = \omega_F(1+\gamma)^{1/2} \tag{4}$$

where

$$\gamma = (K/\rho_m h)/\omega_F{}^2$$

Here, ω_{F_e} is the flutter frequency of the panel supported on elastic foundation, and ω_F is the flutter frequency of the same panel without elastic support. Eq. (3) can be interpreted as a modification of the effective damping of the panel due to the presence of the elastic foundation. Because of the increase in total damping, the flutter velocity of the panel generally increases with the stiffness of the springs (see Fig. 5 of Ref. 6).

In Ref. 6, it was found that for low values of the damping coefficient g_T, the flutter dynamic pressure parameter λ_F is independent of g_T; for high values of g_T, λ_F is roughly proportional to it. A similar interpretation may also be given for g_{T_e}. This means that for a low value of g_T, λ_F increases directly with g_{T_e}. This also suggests that for low values of g_{T_e}, it is possible to use the quasi-static (Ackeret) approximation for aerodynamic loading with reasonable accuracy. Further, it can also be found from Ref. 6 that flutter modes change from standing waves at low values of g_{T_e} to traveling waves at high values of g_{T_e}. Hence, the presence of springs not only affects the flutter velocity and the flutter frequency of the panel, but can also change the type of flutter mode, depending upon the support stiffness.

Some comment may be made concerning the flutter of two parallel panels joined by an elastic medium, a problem discussed in Refs. 4 and 5. Here, because of the coupling of the motion of the two plates in the presence of elastic springs, the dynamic characteristics of this composite system are quite different from those of individual panels. Depending upon the inplane forces, the flutter characteristics of this configuration are very much affected by the stiffness of the elastic medium, regardless of whether or not damping is considered.

Conclusion

Using a simple analogy, the flutter characteristics of a uniform panel resting on a linear elastic foundation can be easily obtained from those of a panel of the same geometry with no elastic support. Using quasi-steady aerodynamic loading, it may be observed that there is not only a change in the flutter frequency, but also of the total damping of the system, due to the elastic foundation. Further, it is found that for small values of aerodynamic damping and spring stiffness, the quasi-static approach is reasonable, whereas for high values of aerodynamic damping or of spring stiffness, quasi-static theory will not be adequate. Also, for high values of spring stiffness the flutter mode of the panel exhibits traveling wave motion.

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Laminar Separated Flow over Nonlifting Ellipses

Nathan Ness*

West Virginia University, Morgantown, West Va.

FIGURES 1 and 2 present the results of calculations for the laminar separated flow over nonlifting ellipses as functions of the thickness/chord ratio t/c and the freestream Reynolds number Re_∞ ($= V_\infty c/v$). The results include the laminar separated flow over the circular cylinder as the special case when $(t/c) = 1$. The calculations involve a coupling of inviscid and viscid flow, i.e., the inviscid analysis provides the pressure which is used in the boundary-layer program to locate the boundary-layer separation points.

The inviscid calculations are based on the work of Parkinson and Jandali[1] who determined the effect of the wake on several symmetrical nonlifting blunt bodies. Their theoretical model provides an expression for the pressure distribution along the wetted surface PLQ of the body (Fig. 1), an expression for the asymptotic wake height H, and an expression for the pressure

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* Professor of Aerospace Engineering. Associate Fellow AIAA.